AQA

A-LEVEL
Mathematics

MS04 – Statistics 4
Mark scheme

6360
June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS04

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

GN8 Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.

Q	Solution	Mark	Total	Comment
1	Assumption: **differences** are (approximately) **normally** distributed	B1		OE; must mention 'differences'
	d: 3.7 –2.8 –5.8 –3.0 –3.5 4.2 **or** d: –3.7 2.8 5.8 3.0 3.5 –4.2	M1		
	$\bar{d} = \pm 1.2$ and $s_{n-1}^2 = 17.08$ or $s_{n-1} = 4.13$ or $s_n^2 = 14.24$ or $s_n = 3.77$	A1		CAO/AWRT (17.08040 & 4.13328) (14.23667 & 3.77315)
	CV: 90% $\Rightarrow$ $t_5(0.95)$ = **<u>2.01 to 2.02</u>**	B1		AWFW (2.015048)
	CI: $(\pm 1.2) \pm (2.015) \times \left(\frac{4.13}{\sqrt{6}}\right)$	M1		Must use $\left(\frac{4.13}{\sqrt{6}}\right)$ or $\left(\frac{3.77}{\sqrt{5}}\right)$ OE
	Thus **<u>–1.2 ± 3.4 or +1.2 ± 3.4</u>** **or** **<u>(–4.6, 2.2) or (–2.2, 4.6)</u>**	A1		AWRT
			6	
Note	**1** CI based on two independent samples $\Rightarrow$ B1 M0 A0 B0 M0 A0 (max of 1 mark)			
		Total	**6**	

Q	Solution	Mark	Total	Comment
2 **(a)**	$\sum(x-\bar{x})^2 = 762 \quad s_{n-1}^2 = 50.8 \quad s_{n-1} = 7.12741$ $s_n^2 = 47.625 \quad s_n = 6.90109$	B1		CAO/AWRT; any one Ignore notation
	$H_0 : \sigma = 10$	B1		OE
	$H_1 : \sigma \neq 10$	B1		OE
	DF $\quad \nu$ = **<u>15</u>**	B1		CAO; can be implied
	CVs $\quad \chi^2(0.95)$ = **<u>6.26 and 27.5</u>**	B1		AWRT; both (6.262 & 27.488)
	$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{762}{100} =$ **<u>7.6 to 7.65</u>**	M1 A1		AWFW (7.62) (p-value = 0.123774)
	There is **no evidence**, at 5% level, of a **change from 10** in the standard deviation	AF1	**8**	OE; F on χ^2-value and both χ^2-CVs **or** on correct use of p-value
(b)	**No significant evidence** of change in σ **or standard deviation** is **known/same** so use a ***z*-test**	Bdep1 B1		OE: dep on "Accept H_0" in (a)
	or **Some numerical evidence** of change/reduction in σ from 10 (eg 7.12 < 10) so use a ***t*-test**	 (Bdep1) (B1)	**2**	OE; dep on $s_{n-1} < 10$ or $s_n < 10$
		Total	**10**	

Q	Solution	Mark	Total	Comment
3 (a)(i)	If $\frac{1}{\lambda}$ is mean, then $\mathbf{f(x) = \lambda e^{-\lambda x}}$	M1		Correct pdf identified or used (Given in blue booklet)
	so $F(m) = \int_0^m \lambda e^{-\lambda x} dx = 0.5 \Rightarrow$	m1		Correct expression equated to 0.5 but ignore limits
	$\left[-e^{-\lambda x}\right]_0^m = 0.5 \Rightarrow$ $1 - e^{-\lambda m} = 0.5$ **or** $e^{-\lambda m} = 0.5 \Rightarrow$	A1		Correct integration and correct substitution of correct limits
	$\mathbf{m = \frac{-\ln 0.5}{\lambda}}$ **or** $\mathbf{m = \frac{1}{\lambda}\ln 2}$ **or** $\mathbf{m = \frac{0.693}{\lambda}}$	A1	**4**	OE CAO/AWRT (Use of $\mu = \lambda \Rightarrow$ M1 m1 max)
(ii)	$P(m < X < \mu) = P\left(X < \frac{1}{\lambda}\right) - 0.5 \Rightarrow$	M1		Use of $\left(\frac{1}{\lambda}\right)$ & (0.5 OE)
	$\left[-e^{-\lambda x}\right]_0^{\frac{1}{\lambda}}$ or $F\left(\frac{1}{\lambda}\right) = 1 - e^{-1} - 0.5 \Rightarrow$	A1		Correct integration and correct substitution of correct limits or correct use of $F\left(\lambda^{-1}\right)$
	$\mathbf{0.5 - e^{-1}}$ **or** $\mathbf{\frac{e-2}{2e}}$ **or** **0.132**	A1	**3**	OE CAO/AWRT (0.132121)
(b) (i)	$E(Y) = \frac{1}{0.0125} = 80$			(From blue booklet)
	$P(Y < 2E(Y)) = \left[-e^{-0.0125y}\right]_0^{160} =$	M1		Use of F(160); OE
	$\mathbf{1 - e^{-2}}$ **or** $\mathbf{\frac{e^2 - 1}{e^2}}$ **or** **0.865**	A1	**2**	OE CAO/AWRT (0.864665)
(ii)	No calls during 2019 $\Rightarrow$ $Y >$ **365**	B1		CAO
	$P(Y > 365) = \left[-e^{-0.0125y}\right]_{365}^{\infty} =$	M1		Use of 1 – F(365); OE
	$\mathbf{e^{-4.5625}}$ **or** **0.010 to 0.011**	A1	**3**	CAO/AWFW (0.0104359)
		Total	**12**	

<table>
<tr><th>Q</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>4
(a)</td><td>Mean = $\frac{\mathbf{300}}{\mathbf{125}}$
= <u>2.4</u></td><td>B1</td><td>1</td><td>CAO ratio
AG</td></tr>
<tr><td>(b)</td><td>H_0 : claim justified or Poisson
H_1 : claim not justified or not Poisson

<table><tr><th>n</th><th>p</th><th>E</th><th>O</th><th>$(O-E)^2/E$</th><th>E</th><th>O</th><th>$(O-E)^2/E$</th></tr><tr><td>0</td><td>0.09071</td><td>11.34</td><td>14</td><td>0.62408</td><td>11.34</td><td>14</td><td>0.62408</td></tr><tr><td>1</td><td>0.21772</td><td>27.22</td><td>23</td><td>0.65292</td><td>27.22</td><td>23</td><td>0.65292</td></tr><tr><td>2</td><td>0.26127</td><td>32.66</td><td>35</td><td>0.16788</td><td>32.66</td><td>35</td><td>0.16788</td></tr><tr><td>3</td><td>0.20901</td><td>26.13</td><td>23</td><td>0.37420</td><td>26.13</td><td>23</td><td>0.37420</td></tr><tr><td>4</td><td>0.12541</td><td>15.68</td><td>16</td><td>0.00669</td><td>15.68</td><td>16</td><td>0.00669</td></tr><tr><td>5</td><td>0.06020</td><td>7.52</td><td>10</td><td>0.81441</td><td>11.98</td><td>14</td><td>0.33930</td></tr><tr><td>6</td><td>0.02408</td><td>3.01</td><td>4</td><td>0.32576</td><td></td><td></td><td></td></tr><tr><td>≥7</td><td>0.01160</td><td>1.45</td><td>0</td><td>1.44926</td><td></td><td></td><td></td></tr><tr><td>Total</td><td>1</td><td>125</td><td>125</td><td>4.415</td><td>125</td><td>125</td><td>2.165</td></tr></table>
χ^2(calc) = <u>2.0 to 2.3</u>

DF $\nu = 6 - 1 - 1 =$ <u>4</u>

CV $\chi^2(0.95)$ = = <u>9.48 to 9.49</u>

No evidence, at 5% level, to suggest that claim is incorrect</td><td>B1

M1

A1

M1

M1

M1

A1

B1

B1

Adep1</td><td>(6)

10</td><td>OE; at least H_0

Use of Po(2.4) to obtain probabilities

≥5 p (3dp) or E (1dp); can be implied

$125 \times p$

Combining outcomes

Use of $(O - E)^2/E$ (OE)

AWFW (2.16508)

CAO

AWFW (9.48773)
(p-value = 0.70 to 0.71 AWFW)

Dep on correct χ^2-value and correct χ^2-CV or on correct use of p-value</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Total</td><td>11</td><td></td></tr>
</table>

Q	Solution	Mark	Total	Comment
5				
(a)	$H_0 : \sigma_R^2 = \sigma_U^2$ $H_1 : \sigma_R^2 \neq \sigma_U^2$	B1		Both; allow alternative subscripts
	DF $\quad \nu_1 = \nu_R = \underline{\mathbf{10}} \qquad \nu_2 = \nu_U = \underline{\mathbf{20}}$	B1		CAO; both (allow switched)
	CV (5%, 2-tailed) $\quad \underline{\boldsymbol{F = 2.77 \text{ to } 2.78}}$ **or** $\quad \underline{\boldsymbol{F = 0.36}}$	B1		AWFW (2.77367) AWRT (0.360533)
	$F(\text{calc}) = 51.40/32.40$	M1		Ratio of given sample variances
	$= \underline{\mathbf{1.58 \text{ to } 1.59 \text{ or } 0.63}}$	A1		AWFW/AWRT (1.58642 or 0.63035) (p-value = 0.364 AWRT)
	$\underline{\mathbf{1.59 < 2.77}}$ **or** $\underline{\mathbf{0.63 > 0.36}} \Rightarrow$ Accept H_0	Adep1	**6**	**AG** OE; dep on previous 5 marks
(b)(i)	$s_p^2 = \dfrac{10 \times 51.4 + 20 \times 32.4}{11 + 21 - 2} = \dfrac{1162}{30} \Rightarrow$	M1		Attempt at pooling of given sample variances
	$\underline{\mathbf{38.73 \text{ or } (\sqrt{38.73} = 6.22)}}$	A1		AWRT; either (38.73333 or 6.22361)
	$\nu = 30$ so 95% $\Rightarrow$ $\underline{\boldsymbol{t = 2.04}}$	B1		AWRT (2.042272)
	CI for $\mu_R - \mu_U$ is $(36 - 32) \pm 2.042\sqrt{38.73\left(\dfrac{1}{11} + \dfrac{1}{21}\right)}$	M2 (-1 ee)		OE
	$\underline{\mathbf{4 \pm 4.7}}$ **or** $\underline{\mathbf{(-0.7, 8.7)}}$	Adep1	**6**	CAO Dep on M2 (4 ± 4.73070) CAO
(ii)	Since **0 ε CI** there is **no significant evidence of a difference in mean durations** of walks	Bdep1 Bdep1	**2**	OE; dep on 0 ε CI but not on CI Dep on Bdep1; must mention 'mean'
		Total	**14**	

Q	Solution	Mark	Total	Comment
6(a) **(i)**	$\mathrm{E}(X)=\sum_{x=1}^{\infty} x\,p(1-p)^{x-1}$	M1		Ignore limits; can be implied
	$= p\left(1+2(1-p)+3(1-p)^2+4(1-p)^3+\ldots\right)$	A1		Common factor & series
	$= p\times\dfrac{1}{(1-(1-p))^2} = \underline{\boldsymbol{\dfrac{1}{p}}}$	A1		**AG** Convincing fully correct proof
			3	
Note	**1** $\mathrm{E}(X)-q\mathrm{E}(X) = p+pq+pq^2+pq^3+\ldots = 1 \Rightarrow \mathrm{E}(X) = 1/p$			
(ii)	$\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$	M1		Other methods are possible
	$= \mathrm{E}(X(X-1))+\mathrm{E}(X)-(\mathrm{E}(X))^2 =$			$\mathrm{E}(X^2)=(2-p)/p^2$
	$\dfrac{2(1-p)}{p^2}+\dfrac{1}{p}-\dfrac{1}{p^2} = \dfrac{2-2p+p-1}{p^2} = \underline{\boldsymbol{\dfrac{1-p}{p^2}}}$	A1		Clear fully correct proof
			2	
(iii)	$\sum_{x=n}^{\infty}\mathrm{P}(X=x)=\sum_{x=n}^{\infty}p(1-p)^{x-1} =$			Or $= \mathrm{P}((n-1)\text{ successive failures}) = q^{n-1}$ $\Rightarrow$ M1 A1
	$p(1-p)^{n-1}\left(1+(1-p)+(1-p)^2+(1-p)^3+\ldots\right)$	M1		Common factor & series
	$= p(1-p)^{n-1}\times\dfrac{1}{1-(1-p)} = \underline{\boldsymbol{(1-p)^{n-1}}}$	A1		**AG** Convincing fully correct proof
			2	
Note	**1** $\sum_{x=n}^{\infty}\mathrm{P}(X=x)=\sum_{x=1}^{\infty}pq^{x-1}-\sum_{x=1}^{n-1}pq^{x-1} = 1-p(1-q^{n-1})/p = q^{n-1} = (1-p)^{n-1}$ {using $S_n=a(1-r^n)/(1-r)$ }			
(b)(i)	$Y\sim$ Geo(1/37) $\Rightarrow$			
	$\mathrm{E}(Y) = \underline{\mathbf{37}}$	B1		CAO
	$\mathrm{Var}(Y)=\dfrac{1-\frac{1}{37}}{(\frac{1}{37})^2}$ or $37^2-37 = \underline{\mathbf{1332}}$	B1		CAO
			2	
(ii)	$\mathrm{P}(Y\geq n)<0.01 \Rightarrow \left(1-\dfrac{1}{37}\right)^{n-1}<0.01$	M1		Use of $(1-p)^{n-1}<0.01$
	so			M1 m1 can be scored for use of n and/or 1/36
	$(n-1)\ln\left(\dfrac{36}{37}\right)<\ln(0.01) \Rightarrow$	m1		$(n-1)\log\left(\dfrac{36}{37}\right)<\log(0.01)$
	$n-1>\dfrac{-4.605170}{-0.027400} = 168.07$			$n-1>\dfrac{-2}{-0.011899}$ or $\dfrac{\ln(100)}{\ln(37/36)}$
	Minimum n = **170**	A1		CAO; use of 1/36 gives 165
			3	
		Total	**12**	

Q	Solution	Mark	Total	Comment
7				
(a)	$X \sim \text{Po}(\lambda) \Rightarrow \text{E}(X) = \underline{\boldsymbol{\lambda}}$ and $Y \sim \text{Po}(4\lambda) \Rightarrow \text{E}(Y) = \underline{\boldsymbol{4\lambda}}$			
	$\text{E}(S) = \underline{\boldsymbol{n\lambda}}$ and $\text{E}(T) = \underline{\boldsymbol{8n\lambda}}$	B1		Both
	$\text{E}(U) = \frac{1}{9n}(n\lambda + 8n\lambda)$ (= $\underline{\boldsymbol{\lambda}}$)	B1		Correct expression
	and $\text{E}(V) = \frac{1}{12n}(4n\lambda + 8n\lambda)$ (= $\underline{\boldsymbol{\lambda}}$)	B1	**3**	Correct expression
(b) **(i)**	$\text{Var}(S) = \underline{\boldsymbol{n\lambda}}$ and $\text{Var}(T) = \underline{\boldsymbol{8n\lambda}}$	B1		Both; can be implied below
	$\text{Var}(U) = \frac{1}{9^2 n^2}(n\lambda + 8n\lambda) = \underline{\frac{\boldsymbol{\lambda}}{\boldsymbol{9n}}}$	M1 A1		≥1 correct use of 'Var' operator CAO
	$\text{Var}(V) = \frac{1}{12^2 n^2}(4^2 n\lambda + 8n\lambda) = \frac{24\lambda}{144n} = \underline{\frac{\boldsymbol{\lambda}}{\boldsymbol{6n}}}$	A1	**4**	CAO
(ii)	$\text{Var}(U)$ and $\text{Var}(V) \to 0$ as $n \to \infty$	BF1	**1**	F on (b)(i)
(c)	Efficiency of U relative to V is			
	$\frac{1/\text{Var}(U)}{1/\text{Var}(V)} = \frac{9n/\lambda}{6n/\lambda}$	M1		
	$= \underline{\textbf{3/2 or 1.5}}$	A1	**2**	CAO
SC	**1** Efficiency = 2/3 or 0.67(AWRT) $\Rightarrow$ B1			
		Total	**10**	